

The 2+1 flavor topological susceptibility from the asqtad action at 0.06 fm

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We report new data for the topological susceptibility computed on 2+1 flavor dynamical configurations with lattice spacing 0.06 fm, generated with the asqtad action. The topological susceptibility is computed by HYP smearing and compared with rooted staggered chiral perturbation theory as the pion mass goes to zero. At 0.06 fm, the raw data is already quite close to the continuum extrapolated values obtained from coarser lattices. These results provide a further test of the asqtad action with rooted staggered flavors.

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1. Introduction

In 2001 S. Dürr presented [1] an analysis of the dependence of the topological susceptibility on the pion mass, as measured in then current full QCD simulations. His comparison included results using Wilson fermions from CP-PACS, UKQCD, and SESAM/T χ L, as well as results using thin link staggered fermions from the Pisa group, and by A. Hasenfratz who analyzed MILC and Columbia dynamical lattices. The conclusion from these studies was that simulations were not yet in agreement with chiral perturbation theory [4] which says that (using $f_\pi = 130$ MeV)

$$\chi_{\text{topo}} \sim \frac{f_\pi^2 m_\pi^2}{4N_f}$$

as the pion mass tends to zero. While in most simulations there was a reduction in χ_{topo} as m_π^2 is reduced, contact with the above line was largely absent. This is displayed here in figure 1, reproduced from [1], with the above expectation (linear in m_π^2) shown as the left black line, against the data. Also shown is the quenched $m \rightarrow \infty$ expectation as a horizontal line on the right.

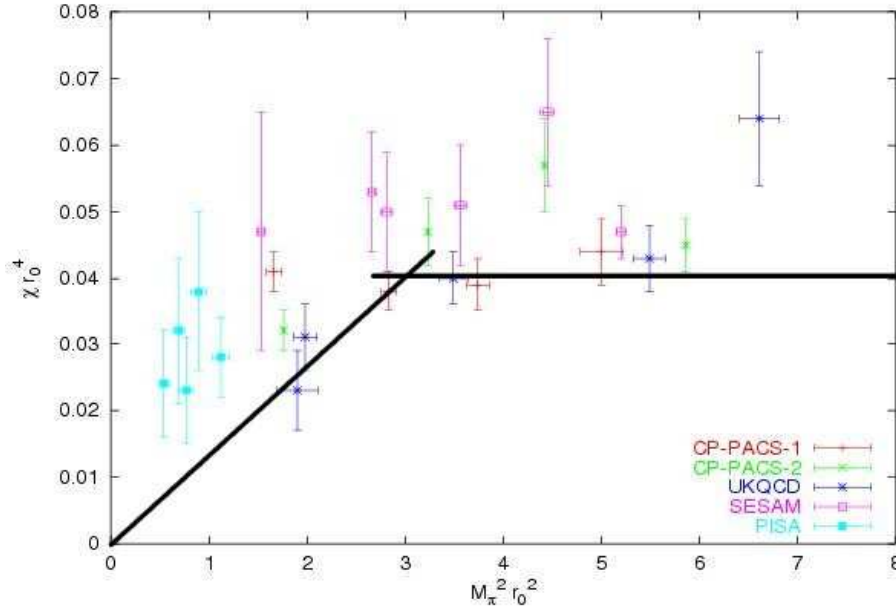


Figure 1: Comparison of full QCD data for χ_{topo} ca. 2001, taken from [1].

About the same time, improved actions were seeing a renaissance and have since generated striking results in almost all areas of lattice gauge theory.

In this contribution we present the latest results of the MILC collaboration for the topological susceptibility and its mass dependence as $m \rightarrow 0$, using the Asqtad action [2] at a lattice spacing of 0.06 fm. With this improved action, HYP smearing, a variance reduction technique to determine the topological charge, and careful extrapolation to the continuum limit using rooted staggered chiral perturbation theory, we indeed see encouraging agreement with the expectations from QCD.

2. Simulations

By 2003 the MILC Collaboration had lattices at two lattice spacings, $a = 0.12$ and $a = 0.09$ fm, and a variety of masses with which to investigate the dependence of the topological susceptibility on quark (or equivalently pion) mass; these results were presented in [3] and are shown below in figure 2.

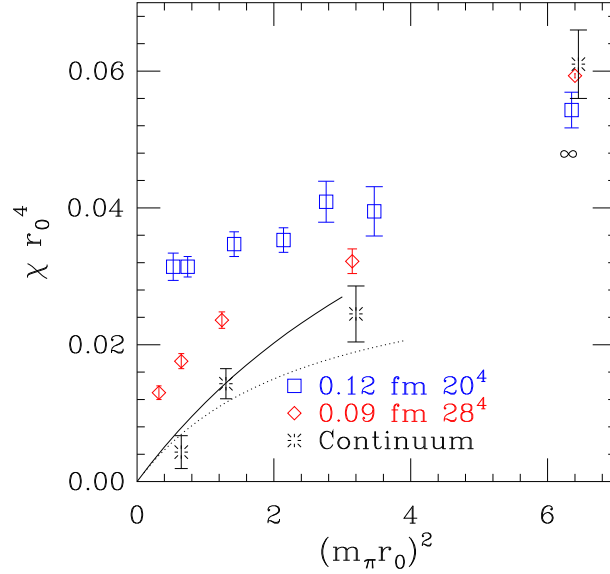


Figure 2: MILC data from [3] (2003).

This result was quite encouraging, as the expectations of chiral perturbation theory appear to be supported by the data.

In the last 4 years, much larger lattices (up to $48^3 \times 144$) at a lattice spacing of 0.06 fm have been generated and their topological charge analyzed. It is the purpose of this note to update the above picture with the new results. In addition to now having three lattice spacings, there have also been some theoretical developments in the methods used to extrapolate continuum values which we will report as well.

2.1 Topological charge and susceptibility measurements

As in [3], we continue to measure the topological charge density $F_{\mu\nu}\tilde{F}^{\mu\nu}$, using the Boulder extended link definition and three HYP smearing sweeps [5].

Typically the topological susceptibility, $\chi_{\text{topo}} = \langle Q^2 \rangle / V$, is computed by averaging the individual Q^2 from each lattice in the ensemble (V is the lattice volume). For our study, we have used a method introduced in [6] which significantly reduces the variance in Q^2 . Since

$$\chi_{\text{topo}} = \frac{\langle Q^2 \rangle}{V} = \int dr \langle q(r)q(0) \rangle$$

we measure the correlator $\langle q(r)q(0) \rangle$, which we split into a short distance ($r < r_{\text{cut}}$) part and a long distance part ($r > r_{\text{cut}}$). r_{cut} is ~ 8 -10 in lattice units.

At short distance, where the correlator is large, we use the measured points in computing $\langle q(r)q(0) \rangle$, whereas at long distance since the measured correlator has large variance, we use values obtained by a fit to points in the transition region (using the green points in the example shown in figure 3). The fit is to a Euclidean scalar propagator, $mK_1(mr)/4\pi^2 r$, which is the expected long distance behavior of $\langle q(r)q(0) \rangle$.

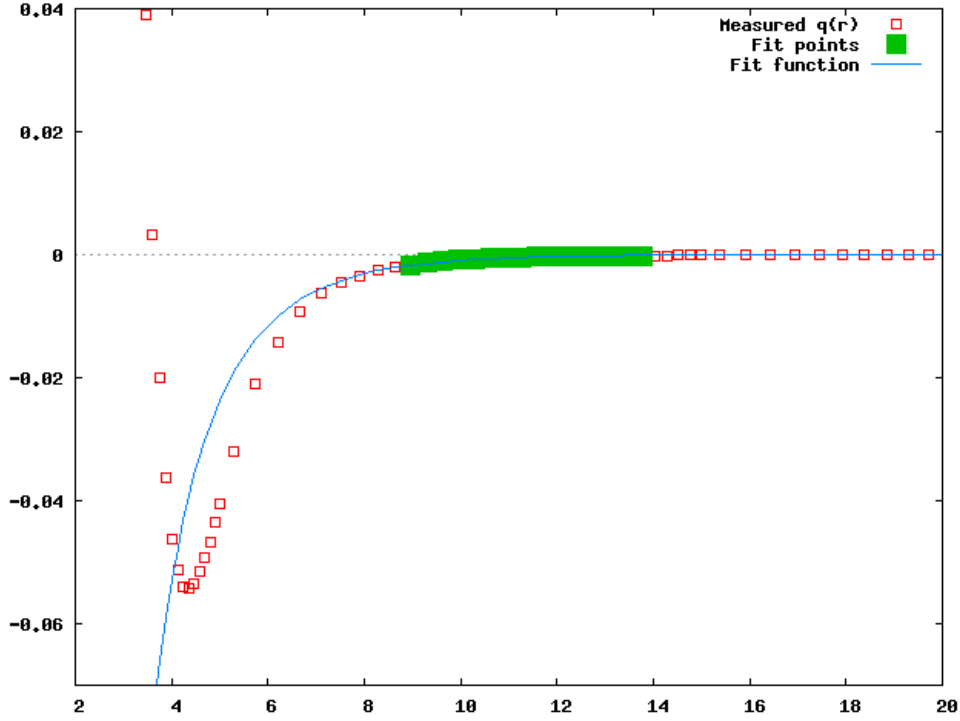


Figure 3: Points used to compute $\langle q(r)q(0) \rangle$. Measured points at $r < r_{\text{cut}} \sim 9a$ are used. For $r > r_{\text{cut}}$ values from a fit to those points in green are used.

2.2 Staggered chiral perturbation theory

In addition to finer lattices and a variance reduction method for our χ_{topo} measurement, there have also been improvements to the expectations from chiral perturbation theory. In particular there is now good analytic understanding of the taste splittings induced in operators at finite lattice spacing. [7, 8]. An example of these taste splittings in the staggered pion multiplet is shown in figure 4. The masses of the 16 pions of various tastes are shown as the quark mass is taken to zero. At finite lattice spacing the splitting is seen to be roughly constant. As $a \rightarrow 0$ these pions become degenerate. The residual $U(1)$ chiral symmetry of staggered fermions provides one Goldstone pion in the multiplet (shown in the figure above with black diamonds), whose mass goes to zero with vanishing quark mass, even at nonzero lattice spacing. Typically it has been this pion which is studied in lattice QCD, as was the case in our previous topological susceptibility study [3].

In [9] however, it was recognized that the staggered pion field which couples to the anomaly, and hence should be used for the relevant chiral perturbation theory, is the *taste singlet* pion,

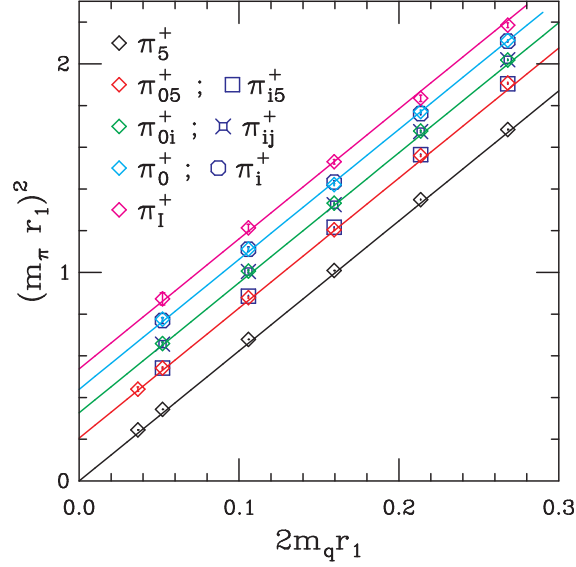


Figure 4: Taste splittings of the staggered pion taste multiplet as a function of quark mass

with mass $m_{\pi,I}$, as opposed to the commonly used Goldstone pion (a pseudoscalar in taste). This taste singlet pion is the one displayed with magenta diamonds in figure 4, at the top of the multiplet.

Following [7, 8] and starting from the rooted staggered chiral lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{8} \text{Tr}(\partial_\mu U^\dagger \partial_\mu U) - \frac{\mu f_\pi^2}{4} \text{Tr}[\mathcal{M}(U^\dagger + U)] + \frac{m_0^2}{2} \phi_{0I}^2 + \sum C_i \mathcal{O}_i + \dots$$

where $\frac{m_0^2}{2} \phi_{0I}^2$ is an explicit mass term representing the coupling of the anomaly to the taste singlet pion field ϕ_{0I} , Billeter, Detar, and Osborn derive [9] the following dependence of χ_{topo} on pion masses in the 2+1 flavor case:

$$\chi_{\text{topo}} = \frac{f_\pi^2 m_{\pi,I}^2 / 8}{1 + m_{\pi,I}^2 / (2m_{\bar{s}s,I}) + 3m_{\pi,I}^2 / (2m_0^2)} \quad (2.1)$$

where $m_{\bar{s}s,I}$ is the $\bar{s}s$ taste singlet pseudoscalar meson mass. This formula interpolates smoothly between the $m_{\pi,I}^2 \rightarrow 0$ chiral limit:

$$\lim_{m \rightarrow 0} \chi_{\text{topo}} \sim \frac{f_\pi^2 m_{\pi,I}^2}{8}$$

and the quenched limit:

$$\lim_{m \rightarrow \infty} \chi_{\text{topo}} \sim \frac{f_\pi^2 m_0^2}{12} \approx 0.06 / r_0^4$$

In the last formula, we use the measured quenched topological susceptibility to set the value of m_0 , and we repeat that $m_{\pi,I}$ here is the mass of the taste-singlet pion.

$am_{u,d} / am_s$	$10/g^2$	lattice spacing	$L^3 \times T$	# lats.
quenched	8.00	a = 0.12 fm	$20^3 \times 64$	408
0.05 / 0.05	6.85			425
0.04 / 0.05	6.83			351
0.03 / 0.05	6.81			564
0.02 / 0.05	6.79			484
0.01 / 0.05	6.76			658
0.007 / 0.05	6.76			493
quenched	8.40	a = 0.09 fm	$28^3 \times 96$	396
0.031 / 0.031	7.18			496
0.0124 / 0.031	7.11			527
0.0062 / 0.031	7.09			592
0.0072 / 0.018	7.48	a = 0.06 fm	$48^3 \times 144$	624
0.0036 / 0.018	7.47			608

Table 1: Lattices and parameters used in this study

3. Results

Putting these developments together, we present our latest results for the 2+1 flavor topological susceptibility. The lattices used for this study have taken more than five years to produce and analyze, and are shown in Table 1. Having χ_{topo} at numerous quark masses and three lattice spacings, we fit our entire data set to an interpolating function in lattice spacing, a , and taste singlet pion mass squared, $m_{\pi,I}^2$ (the strange taste-singlet mass $m_{\bar{s}s,I}$ on these lattices was tuned to be constant):

$$\frac{1}{\chi_{\text{topo}} r_0^4}(m_{\pi,I}^2, a) = A_0 + (A_1 + A_2 a^2 + A_3 a^4)/m_{\pi,I}^2.$$

The continuum limit is obtained from the fit by setting $a = 0$ in this function, and we are left with $\chi_{\text{topo}}^{\text{cont.}}(m_{\pi,I}^2)$ extracted from our data. The result is shown below in figure 5. Measured lattice data are shown with blue, red, and magenta symbols, while the continuum limit extrapolation function $\chi_{\text{topo}}^{\text{cont.}}(m_{\pi,I}^2)$ is shown with a solid black line. Some representative points along this line are shown with error bars reflecting the errors of the continuum extrapolation. Finally, two functions representing the chiral perturbation prediction of eq. (2.1) are shown in green: the lower line “L.O. 2+1+ m_0 ” includes the value for m_0 set by the quenched data, and eq. (2.1) with $m_0 = \infty$ is shown labeled “L.O. 2+1” for comparison.

4. Conclusions

With the addition of the new $a = 0.06$ fm data, we see that the topological susceptibility is behaving as expected in the $m_{\pi,I}^2 \rightarrow 0$ limit of rooted staggered chiral perturbation theory. We find it striking that the lightest 0.06 fm datum is almost on the continuum line without extrapolation.

Finally, we feel that these results lend further credibility to the use of the “fourth root method” to simulate single flavors. As mentioned in M. Creutz’s talk at this conference, aberrant results

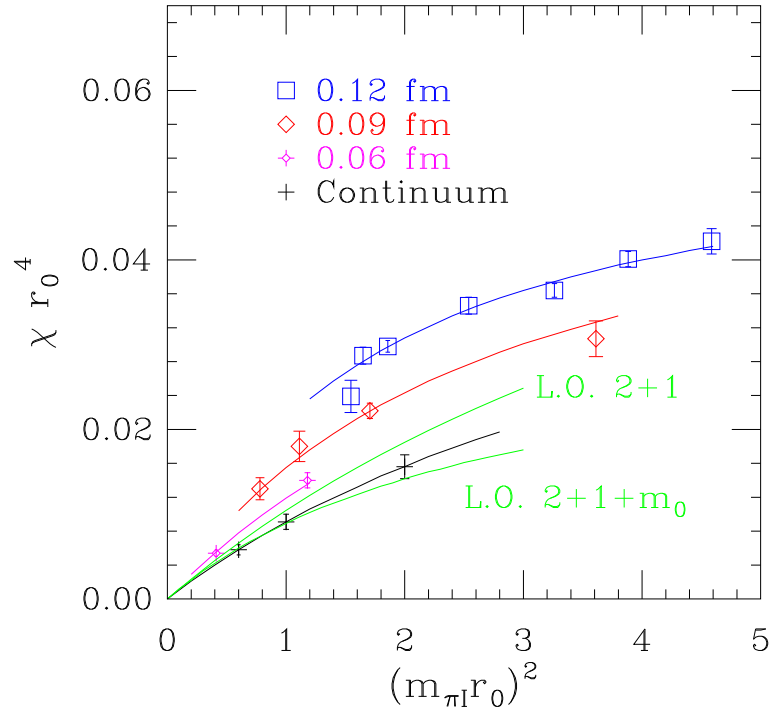


Figure 5: Latest MILC results for the topological susceptibility from lattices down to $a = 0.06$ fm.

from the fourth root would be expected to arise first in violations of topological quantities and correlations, which are quite sensitive to the number of flavors. We see no such violations, and indeed only strong support that the simulations are behaving as expected from QCD.

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